

CRITERIA FOR SIMILARITY IN THE TRANSPORTATION OF SEDIMENT

by

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SIMILITUDE AS A SUBSTITUTE FOR FUNCTIONAL ANALYSIS

Any phenomenon of fluid motion may be described mathematically by means of a functional relationship among a group of dimensionless parameters so constituted as to include the one dependent factor to be studied and all the independent quantities which govern its variation. Were the forms of these functional relationships known for all phases of hydraulics, there would be no need whatever of investigating the behavior of projected engineering works by means of laboratory models. However, in most cases these complex functions are at best only qualitatively understood, and one must generally resort to the principle of similitude as a practical substitute for rigorous or even approximate functional analysis. This principle merely embodies the fact that flow in a model will be dynamically similar to that in its prototype if all dimensionless parameters describing the flow in the model are numerically equal to the corresponding parameters for flow in the prototype. There is obviously no necessity of knowing how the several parameters are functionally interrelated, but for complete similarity it is essential that they include every variable which has an influence upon the fluid motion.

It is already a truism of hydraulic model testing that complete similarity is a goal which may often be approached, but seldom attained so long as one is forced to use the same fluid in the model as that which flows in the prototype. The discrepancies between perfect similarity and that actually obtained become the greater as one begins to deal with the movement of sediment by flowing water. Indeed, sediment transportation is perhaps the most complex problem

of hydraulics, combining as it does all the difficulties of unsteady, non-uniform flow of water in open channels, with temporal change of the boundary configuration. Thus, any river displays not only a general downstream movement of sediment which varies with the seasonal rate of discharge, but phases of local scour and undercutting of banks on the one hand and phases of local deposition on the other. While fundamental research on sediment transportation is still restricted to the movement of bed load and suspended load in steady, uniform flow, model studies must seek to reproduce all major variations taking place in the prototype. It is often necessary, however, to sacrifice even approximate similarity in some respects in order to obtain more satisfactory results in specific phases of the motion; unfortunately, the validity of these results can be ascertained only by conducting model studies at various scales and extrapolating to prototype size, or by eventual comparison with prototype behavior. This unhappy situation will not be improved, it would appear, so long as it remains necessary to regard the grain size of the sediment as a geometrical characteristic subject to the same scale reduction as the other linear dimensions.

For the formulation of more general principles of sediment similarity than now exist, three lines of attack are at hand: (1) One may use as a guide the various empirical equations already available, as in the case of bed-load movement. (2) One may turn to theoretical analyses, still only partially tested experimentally, as in the case of sediment suspension by fluid turbulence. (3) For cases of motion as yet untouched either experimentally or analytically, one may obtain the essential results by a combination of dimensional and physical analysis, supplemented by a systematic experimental check. In the following pages each of these methods will be illustrated.

SEDIMENT CHARACTERISTICS

Parameters governing the transport of sediment by flowing water must necessarily include all effective geometric, flow, and fluid characteristics, and in addition the properties of the sediment itself. The characteristics of the fluid motion have been discussed elsewhere in considerable detail, but those of the sediment warrant amplification at this point. These include the size distribution, shape, and density of the individual grains, and the state of the bed as a whole.

Sediment density needs no further comment. Particle shape will

likewise receive little mention herein, owing to the difficulty of measuring and expressing shape factors in a simple, representative fashion. It must suffice to note that in general the sphericity and roundness of natural sediment grains decrease with decreasing grain size, and that the crushed materials often used in the laboratory possess shape factors quite different from the natural product. The degree of compaction of a sediment bed is only partly dependent upon the sediment characteristics, for a given sediment may display different degrees of compaction for different loads or different manners of deposition. However, it must be assumed at present that all materials under discussion are deposited in the same manner, and that the porosity of the bed depends only upon the other sediment properties.

Grading characteristics of sediments are probably of far more importance in transportation studies than is usually believed, and it is imperative that significant parameters be adopted for the size distribution. Mechanical analyses of size¹ are now customarily plotted in the familiar histogram, with geometric (logarithmic) gradation of nominal diameter (Fig. 1a). As the class intervals become smaller,

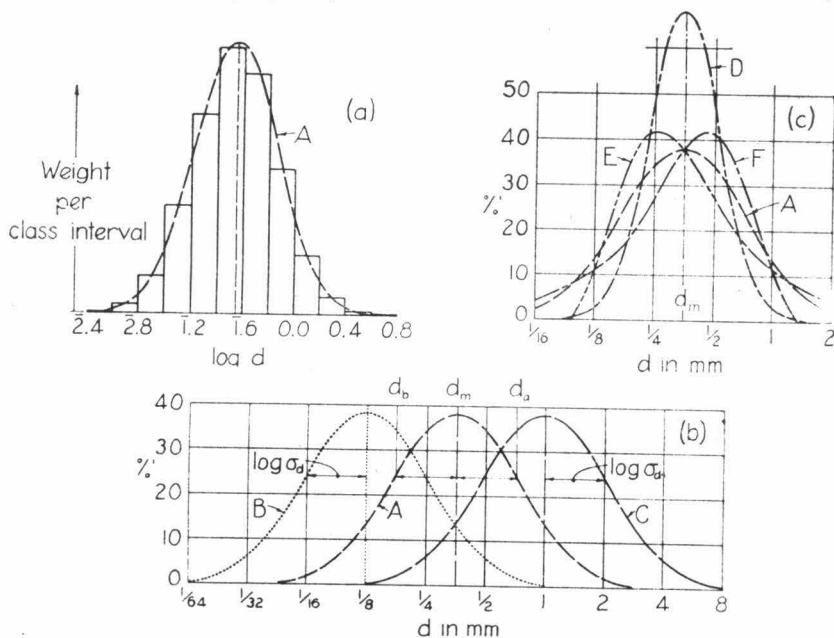


FIG. 1.—DEFINITION SKETCHES FOR THE SIZE-FREQUENCY CHARACTERISTICS OF SEDIMENT.

the histogram approaches the smooth curve of the size-frequency diagram shown as a broken line, the vertical scale of which (per cent per class interval, designated by %' in Fig. 1b) is so adjusted as to make the enclosed area equal to 100%. The geometric mean diameter of the corresponding grains is obtained from the first moment of this area about an arbitrary vertical axis; this primary parameter is indicated by the abscissa of a vertical line passing through the center of gravity of the area under the smooth curve. The standard (root-mean-square) deviation from this mean—i.e., the radius of gyration of the area about the mean line—is obtained from the second moment. Similarly, the skewness (asymmetry) of the curve is obtained from the third moment, kurtosis (foreshortening of the peak) from the fourth moment.

Natural sediments deposited under constant conditions are characterized by size-frequency diagrams approaching the logarithmic form of the normal error curve. Since the error curve possesses neither skewness nor kurtosis, it is evident that it may be fully defined by its geometric mean and by the standard deviation from this mean. Departures of natural sediments from this type of size distribution are of no practical importance to the present discussion, the third and fourth moments henceforth being ignored.

Since the abscissa scale of the size-frequency diagram is logarithmic, the first moment is really the logarithm of the length d_m and the second moment the difference between two logarithms or the logarithm of a length ratio: $\log \sigma_d = \log d_a/d_m = \log d_m/d_b$. Thus, d_m is a measure of the position of the frequency diagram on the size scale, and σ_d a measure of the grading. Obviously, either parameter is independent of the other, sediments having the same proportional range in size being characterized by the same numerical value of σ_d , regardless of the magnitude of d_m . Thus, with reference to Fig. 1, curves A, B, and C represent sediments possessing the same degree of sorting ($\sigma_d=2$) but different mean diameters; curves A and D, the same mean diameters but different degrees of sorting ($\sigma_d=2$, $\sigma_d=\sqrt{2}$); curves E and F, different directions of skewness but the same values of d_m and σ_d as curve A.

Determinations of d_m and σ_d may easily be made from a cumulative plot of the mechanical analysis on logarithmic probability paper. With these coordinates the logarithmic form of the normal error curve becomes a straight line (Fig. 2), its position varying

with d_m , its slope with σ_d . The magnitude of σ_d is given by the ratio of d_m to the intercept at the 15.9 per cent line or the ratio of the 84.1 per cent intercept to d_m . It so happens that many skewed curves (E and F of Figs. 1 and 2) may be approximated in form by two terms of a logarithmic Gram-Charlier series, the latter function invariably coinciding at the 15.9 per cent and the 84.1 per cent points with the normal error function having the same values of d_m and σ_d .

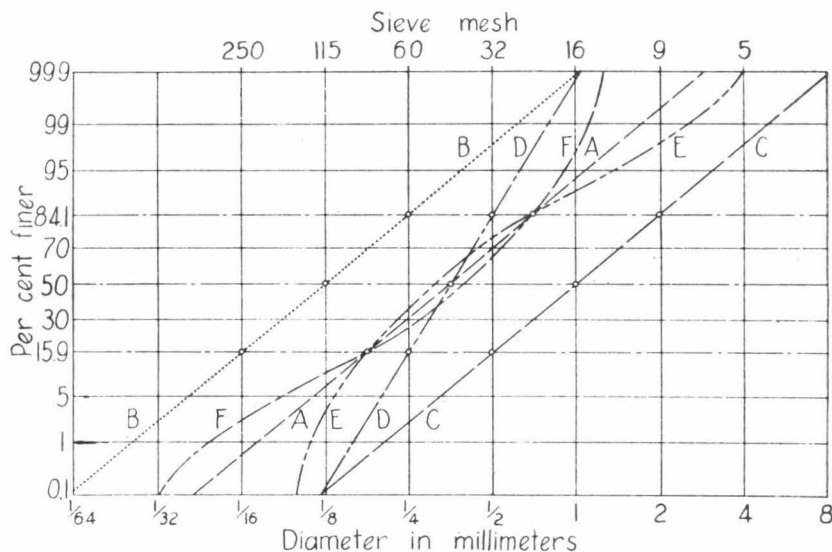


FIG. 2.—CUMULATIVE PROBABILITY PLOTS CORRESPONDING TO THE FREQUENCY DIAGRAMS OF FIG. 1.

The mean and the standard deviation of skewed curves may therefore be obtained with sufficient accuracy for most practical purposes by passing a straight line through the 15.9 per cent and 84.1 per cent on the skewed plot, d_m and σ_d then being found from the straight line as before. Not only are these parameters of great importance in the study of actual sediments found in nature, but they may well serve as two fundamental parameters to be varied systematically in the laboratory study of sediment transportation.

Such characteristics as permeability and angle of repose of the sediment grains cannot be regarded as independent characteristics, for they are governed wholly by the basic properties of sediment and

fluid. Although the fall velocity of the individual grains is likewise a dependent characteristic varying with many factors of sediment and fluid, it is often treated as a basic variable in sediment research. Such use of the fall velocity in place of the other sediment characteristics is permissible when the latter have no individual effect upon the phenomenon, but this substitution should invariably be subjected to careful study before one accepts the simplified mathematical results which the method permits.

SIMILARITY IN BED-LOAD MOVEMENT

The most important advancement in the study of bed load in recent years is Shields' analytical and experimental proof that the initial movement from a level bed of sediment is governed by the ratio of mean sediment diameter to boundary-layer thickness.² Shields reasoned that

$$\frac{T_c}{(\gamma_s - \gamma) d_m} = \phi \left(\frac{d_m}{\delta} \right)$$

the form of the function ϕ being obtained from experiments with crushed amber, lignite, barite, and sand. Shields reasoned that shape and compaction factors should also have an influence upon the motion, but no such influence was detected in the experiments. The effect of σ_d was not studied, all sediments having approximately the same degree of sorting; nevertheless, this factor is probably a very essential characteristic in cases of different size distributions.

A purely dimensional analysis might have yielded the expression

$$\frac{T_c}{\gamma d_m} = \phi \left(\frac{\gamma_s}{\gamma}, \frac{d_m}{\delta}, \sigma_d \right)$$

(not to mention various others of even more complex form), indicating that for similarity of initial movement γ_s/γ , d_m/δ , and σ_d must be numerically identical in model and prototype. Shields' analytical and experimental studies showed, however, that certain of these factors could be combined, so that for sediments of similar grading characteristics the ratio d_m/δ appears to be the sole criterion for similarity of initial movement from a level bed. That d_m must be a basic factor in this case is evident from the fact that the grain size governs the absolute bed roughness so long as the bed remains level. On the

other hand, Shields' experiments indicated that the magnitude of d_m/δ determines as well the form of the initial bed undulations, ripples appearing at small values of d_m/δ and long bars at large values—a point of fundamental importance in the attainment of bed similarity.

Since the rate of bed-load movement has not as yet been subjected to rational analysis, only purely empirical expressions are at hand upon which to base similarity criteria. Any such empirical relationship, it must be noted, is dependable for this purpose only within the range of experimental check, and cannot be generalized with safety. Using Shields' transport equation

$$\frac{G}{Q} = 10 S \frac{T - T_c}{(\gamma_s - \gamma) d_m}$$

for the sake of convenience, it would seem that the ratio of sediment discharge by weight (measured under water) to the water discharge by weight should depend only upon the parameter $S(T - T_c)/(\gamma_s - \gamma) d_m$. Again, however, the expression was developed from studies on uniform materials, and was restricted to cases in which viscous shear at the bed was negligible in comparison with turbulent shear. Therefore, in order for such a criterion to hold, it would seem essential that σ_d be approximately the same in model and prototype and that the Reynolds number be of the same order of magnitude. Needless to say, the Reynolds number in many model studies is far too low to eliminate the effect of viscosity in the boundary region.

While d_m is obviously a primary sediment characteristic in initial phases of bed movement, its rôle in advanced stages of transportation is less apparent—in particular since it is not the grain size, but that of the bed irregularities, which governs the bed roughness. It is a noteworthy fact that all experimental studies of sediment movement have been arbitrarily restricted to cases in which little or no material is carried into suspension—the latter being a phenomenon in which, for the most part, d_m is definitely not a pertinent factor.

SIMILARITY IN THE TRANSPORTATION OF SUSPENDED LOAD

Since the suspension of sediment in flowing water is governed by the turbulence of the flow, it is possible to express the relative form of the sediment-distribution function in terms of the fall-velocity characteristics of the sediment and the distribution of turbulence over the

vertical section; the latter, in turn, is a function of the velocity distribution, while the fall-velocity characteristics may be defined in terms of w_m , the geometric mean, and σ_w , the geometric standard deviation of w from this mean—weight frequencies being used exactly as in the case of the sediment diameter. The distribution of sediment over the vertical section is then a function of the following form:³

$$\phi \left(C_a, \frac{a}{D}, \frac{\sqrt{T/\rho}}{w_m}, \sigma_w \right)$$

Herein, C_a is the weight concentration of the sediment at some arbitrary level a , while $\sqrt{T/\rho}$ is the so-called friction velocity, equal to $V\sqrt{f/8}$. The form of ϕ depends upon the distribution of turbulence, which varies with the velocity distribution, which (for fully developed boundary roughness) depends in turn upon f , or upon the relative roughness k/D , and upon the mean velocity of flow.

Since C_a is an independent variable in this function, the total amount of material transported in suspension is still indeterminate. If one regards the advanced state of sediment movement as the result of fluid turbulence over the entire flow section, it would seem reasonable to replace C_a by C_0 , the bed concentration, and to make a proportional to k , the absolute roughness of the bed. $\sqrt{T/\rho}$ may be replaced by V alone, since f varies only with k/D . The total rate of transport would then have the form

$$\frac{G}{Q} = \phi \left(C_0, \frac{k}{D}, \frac{V}{w_m}, \sigma_w \right)$$

This expression has a reasonable, though hypothetical, basis and has yet to be verified experimentally. Were it to prove valid, the terms C_0 , k/D , V/w_m , and σ_w would be the sole criteria for similarity of sediment discharge under moderate concentrations, provided only that the bed resistance did not depend upon the Reynolds number in either prototype or model.

SIMILARITY IN SCOUR PHENOMENA

Although the phenomenon of localized scour has been subjected to surprisingly little systematic study, it depends upon essentially the same geometric, flow, fluid, and sediment characteristics as any general problem of sediment transportation in unsteady, non-uniform

flow. As in other cases involving the movement of sediment by water, it is not safe practice to combine all pertinent variables by simple dimensional analysis, for the elementary manner of applying the Π -theorem presumes that the problem involves the motion of a homogeneous fluid, which is far from the case; as a result, this method will invariably lead to a parameter of the form d_m/D , making the sediment diameter once and for all a basic geometrical factor subject to exact scale reduction in the model.

As a matter of fact, the most general phase of sediment transportation is a combination of two distinct types of motion—that of a fluid relative to geometrical boundaries, and that of granular solids relative to the moving fluid, the latter motion producing in addition a gradual change in the boundary geometry. The actual function, therefore, must be a composite one, the dimensional analysis of which has not yet been rationally approached. Under such circumstances, one is tempted to assume that the relative motion between sediment and fluid is proportional to the normal settling velocity of the individual sediment grains, this velocity of fall then being used in the customary application of the Π -theorem, and all related sediment characteristics being ignored.

Consider, for instance, a simple case of scour in which such fluid characteristics as weight and viscosity have no effect upon the flow itself, and upon the sediment movement only insofar as they vary the magnitude of w_m . It is desired to express the relative depth of scour s as a function of time, mean velocity of inflow, and the sediment characteristics w_m and σ_w . It follows that

$$\frac{s}{a} = \phi \left(\frac{w_m t}{a}, \frac{V}{w_m}, \sigma_w \right)$$

a being some length characteristic of the boundary geometry. The case is, to all appearances, conveniently simple; however, final acceptance of such a hybrid product of dimensional analysis must remain subject to experimental studies in which each of the individual factors is varied over a considerable range.

EXPERIMENTAL PROCEDURE

In order to test the validity of the foregoing relationship, the boundary conditions shown in Fig. 3 were chosen arbitrarily. Water is fed under constant head from a portable supply system to a small,

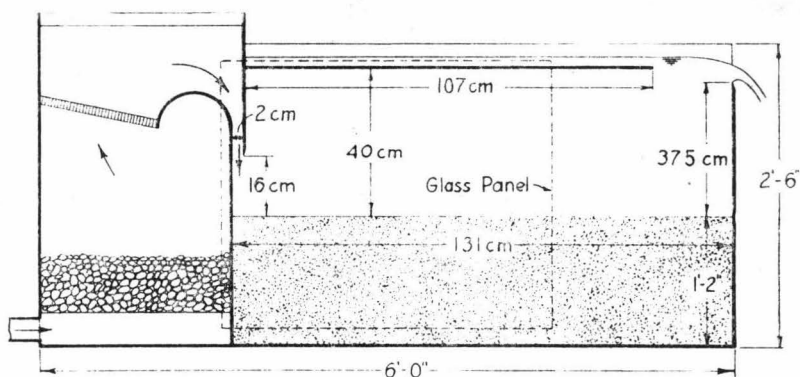


FIG. 3.—LONGITUDINAL SECTION THROUGH GLASS-WALLED TANK, SHOWING THE GEOMETRICAL BOUNDARY CONDITIONS AT THE BEGINNING OF EACH RUN.

glass-walled tank 6 inches in width, the inflow being given the form of a vertical jet impinging on an originally level bed of prepared sand. The course of the scour with time is mapped with wax pencil on one glass panel, the profiles finally being traced full scale on coordinate

paper for permanent reference.

Runs of from 3 to 24 hours duration (depending upon the rate of scour) have been made to date on three different sands of essentially the same σ_w , prepared by a combination of water, air, and sieve classification (sand characteristics are shown

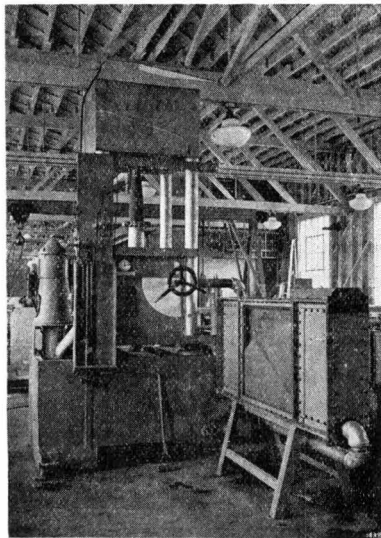


FIG. 4. GENERAL VIEW OF EXPERIMENTAL EQUIPMENT.

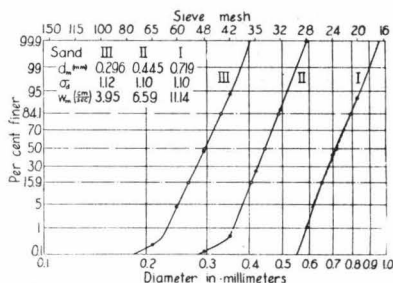


FIG. 5. GRADING CHARACTERISTICS OF SANDS I, II, AND III.

in Fig. 5). In addition, each sand has been studied under a $1/2$ -scale reduction of all geometrical proportions except flume width. Through successive changes in discharge, every series of runs has included as great a variation in the ratio V/w_m as operating conditions would permit.

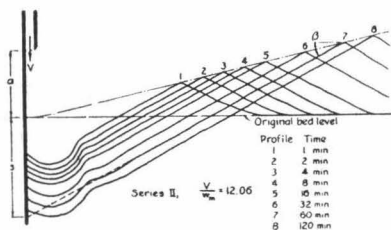


FIG. 6.—PROGRESSIVE DEVELOPMENT OF THE SCOUR PROFILE AS A FUNCTION OF TIME.

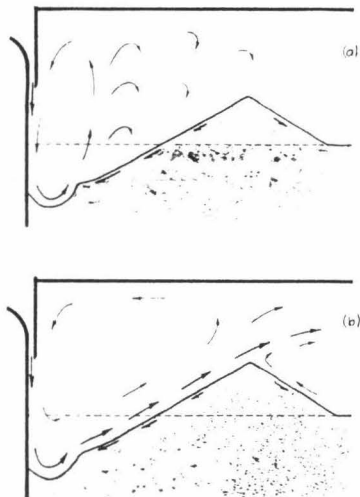


FIG. 7. FLOW PATTERNS FOR (a) MAXIMUM AND (b) MINIMUM JET DEFLECTION.

A family of profiles obtained with Sand II at full scale and constant jet velocity is shown schematically in Fig. 6. Evidently, the linear characteristics of the scour hole vary by approximately constant increments as the successive time intervals double in magnitude, suggesting at once an exponential relation-

ship between scour and time. However, it will be noted that during this particular run the length increments abruptly increase in magnitude, indicating some break in the continuity of the function. Investigation of this discontinuity disclosed the fact that two distinct regimes of flow are possible, the jet either being deflected through nearly 180° (Fig. 7a) or following the boundary of the scour as far as the crest of the dune (Fig. 7b). The depth of scour at which a transition from the former regime to the latter will occur was found to vary with the velocity of the jet, the fall velocity of the material, and the boundary scale, the jet pendulating between its two limiting forms as the

critical stage is approached.

For either regime, the actual removal of material from the scour hole is purely a phenomenon of suspension. Any material falling upon the upstream side of the dune gradually slides back toward the zone of excavation, only to be carried again into suspension as it meets the

deflected jet. In the regime of Fig. 7b (minimum jet deflection) the actual transporting velocities of the fluid are far higher than in the case of Fig. 7a, despite the fact that the velocity of inflow is somewhat lower; as a result, although considerably less material is in suspension, more is carried over the crest before settling out. In select-

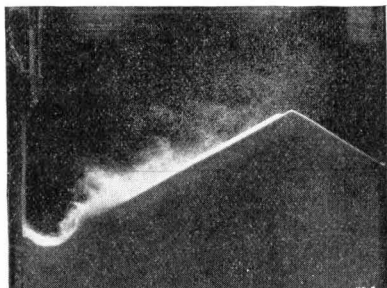
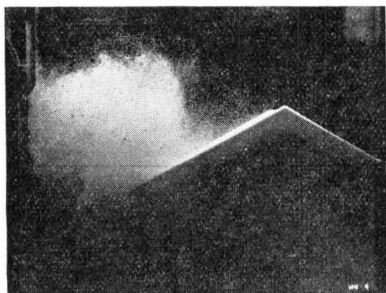


FIG. 8. TYPICAL SCOUR REGIMES AT TWO SLIGHTLY DIFFERENT RATES OF DISCHARGE: ABOVE, MAXIMUM JET DEFLECTION; BELOW, MINIMUM JET DEFLECTION.

ing a characteristic depth of scour s for purposes of comparison, one must therefore distinguish between a factor showing merely the amount of material excavated and one showing the amount carried permanently beyond the excavation zone. The depth of the scour hole itself is of the former category, for at a given position of the dune crest it varies with V , w_m , and the boundary scale. On the other hand, whenever the flow is stopped, all material in suspension settles out and the dune slope adjusts itself to the natural angle of repose of the material—a slope essentially the same as that of the regime shown in Fig. 7b during operation. Accordingly, the variable s was chosen as the intercept on the vertical jet boundary of a sloping line tangent to the face of the dune near its crest (see Fig. 6); its angle of inclination was

slightly greater than 29° for Sands I and II, and somewhat over 30° for the more angular Sand III.

DISCUSSION OF RESULTS

All data obtained thus far with Sands I, II, and III at both full and half scale are reproduced in Fig. 9, s/a being plotted against $\log w_m t/a$ for various values of V/w_m (points shown as squares refer to the regime of maximum jet deflection, circles to that of minimum deflection). At once apparent is the systematic variation of the slope

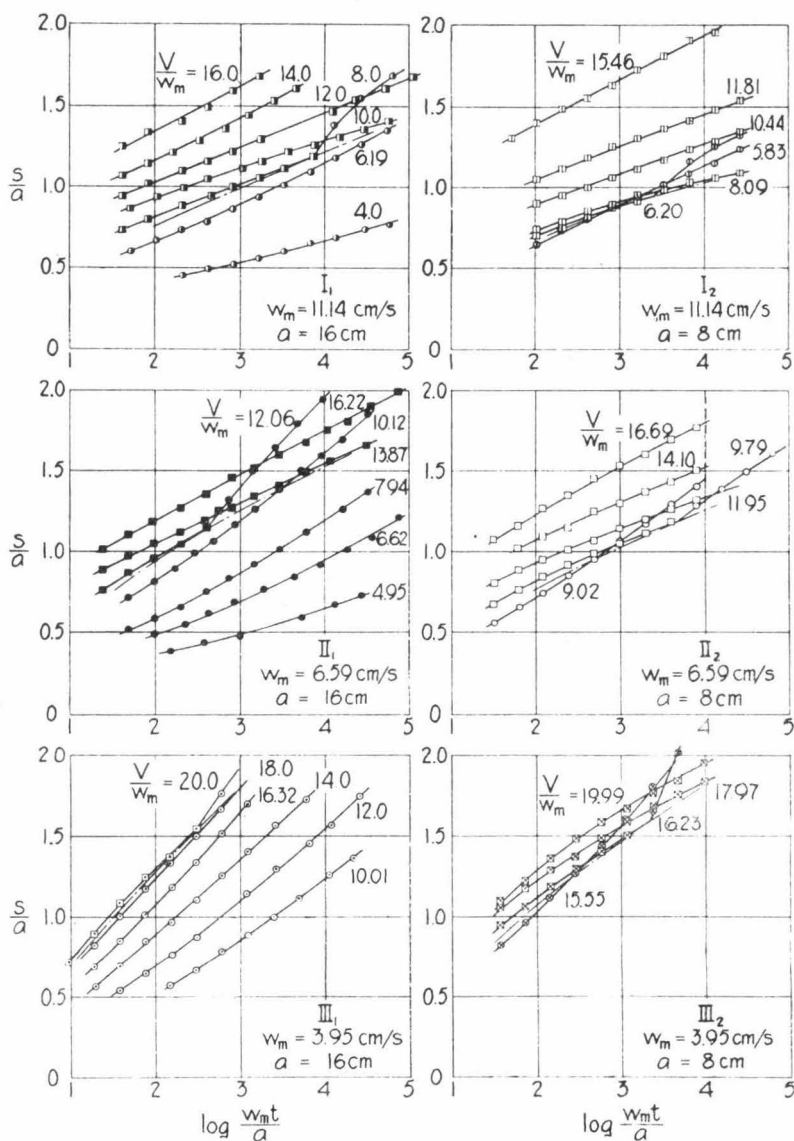


FIG. 9.—EXPERIMENTAL MEASUREMENTS OF LOCALIZED SCOUR.

of the curves of either regime with V/w_m ; in fact, the slope is directly proportional to $V/w_m - 1$, indicating a zero rate of scour for $V/w_m = 1$ —a fact which is quite reasonable, since it is the jet itself, rather than secondary eddies, which accomplishes the scouring action. Further investigation will show that in each regime all curves characterized by the same magnitude of V/w_m will be superimposed if shifted horizontally by an amount proportional to the logarithm of w_m , the constant of proportionality having the value 3.

Such displacement of all curves, followed by multiplication of

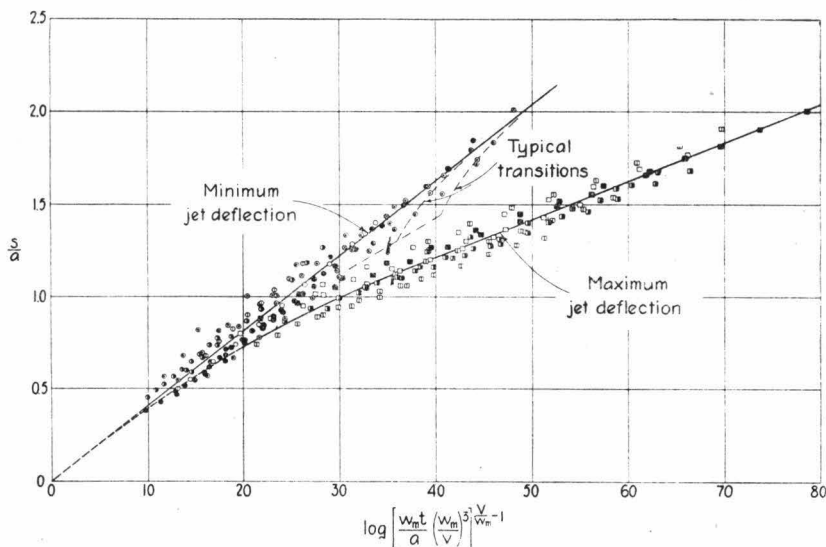


FIG. 10.—COMPOSITE PLOT OF DATA SHOWN IN FIG. 7.

the resulting abscissa values by the quantity $V/w_m - 1$, should then produce a single functional trend for the data of either class. In Fig. 10 is shown such a composite plot of all available measurements for the two distinct regimes (i.e., exclusive of runs in which the transition took place), only the upper two curves of Series III₂ having been eliminated because of unsatisfactory agreement. While a considerable scatter of plotted points is apparent, this condition is typical of all experiments on sediment transportation, and it is felt that the two curves passed through the points indicate the functional trends with good approximation. As a matter of fact, the actual forms of these

functions are of no further concern in the selection of similarity criteria, for a glance at the ordinate and abscissa scales of Fig. 10 will show that the conclusions reached through the dimensional analysis of the problem are fully verified—with one exception: the abscissa term includes a dimensional factor v which was not foreseen in the preliminary analysis. This factor necessarily has the dimension of a velocity, in order that the ratio w_m/v may be properly dimensionless, and it must have the magnitude 5.25 for all three sands if the functional curves are to pass through the origin; while its significance is not fully clear, it might be regarded as the ratio of a to t_o , the latter being some initial time factor dependent upon the model scale—or, perhaps, v is proportional to $\sqrt{p/\rho}$, in which p is a characteristic bearing or shear capacity of the material which tends to oppose the scouring action.

As has already been mentioned, the value of s/a at which the transition from one regime to the other takes place varies with V , w_m , and a , the locus of the transition points for any one series of experiments being indicated in Fig. 9 by a broken line. While further study of the transition phenomenon would be of considerable academic interest, in the organization of similarity criteria it is necessary to define only the approximate transition zone so that model studies may lie entirely within the proper flow regime. This zone is shown in Fig. 11 for each

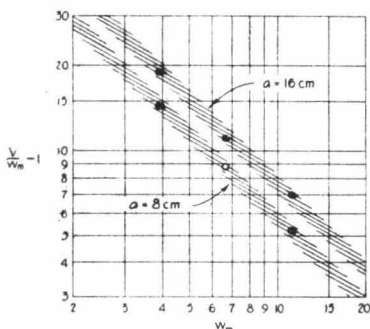


FIG. 11. APPROXIMATE LIMITS OF THE TRANSITION ZONE.

of the two scales already investigated, the points being obtained from Fig. 9 and the width of the shaded areas from exploratory tests for each of the sands and boundary scales.

As yet no detailed investigation has been made of the effect of sediment grading. From Fig. 5 it was seen that the standard deviation of particle size was essentially the same for all three sands, the assumption being made that with such narrow sorting σ_w could then be treated as a constant. For material so nearly uniform in fall velocity, the experiments have shown that no equilibrium of scour is to be expected at any depth, the removal of material continuing as an

exponential function of the time with only the fixed boundaries governing the ultimate limit of excavation. Preliminary experiments indicate, on the other hand, that a process of selective sorting will take place if σ_w is increased appreciably in magnitude, the finer material being carried over the dune, and the coarser being deposited repeatedly on the upstream slope and sliding back into the scour hole. In this way the magnitude of w_m for the material lining the hole will steadily rise, the resulting tendency to approach a state of equilibrium obviously increasing with the magnitude of σ_w for the original bed material.

Whatever the rôle of σ_w may be, the present experiments with constant σ_w indicate that for a given fall-velocity distribution of sediment the sole criterion for similarity of the scour pattern under the existing boundary conditions is embodied in the quantity

$$\left[\frac{w_m t}{a} \left(\frac{w_m}{v} \right)^3 \right]^{\frac{V}{w_m} - 1}$$

provided only that the regime of flow is the same in both prototype and model. Although the mathematical form of this parametric group will probably vary with the boundary conditions, each of the dimensionless ratios should still play an essential part in the phenomenon.

CONCLUSIONS

In selecting criteria for similarity in the transportation of sediment, one must choose, in addition to the usual geometric, flow, and fluid parameters, those sediment characteristics having a definite influence upon the rate at which material is moved. So long as the size of the material relative to some length parameter of the flow is a governing factor, it is only reasonable to use the size-frequency properties as similarity criteria, the geometric mean diameter and the geometric standard deviation from this mean being the most significant size factors available. Thus, when the mean diameter is of the order of magnitude of the boundary-layer thickness, the ratio d_m/δ is a fundamental similarity parameter. Once d_m becomes sufficiently large to produce boundary turbulence, however, neither does viscosity directly influence sediment movement at the boundary nor does the size of moving grains determine the relative roughness of the bed. On the other

hand, it must be noted that bed materials of pulverized pumice, amber, and lignite often used in the laboratory not only far exceed in size the nominal boundary-layer thickness, but often approach the depth of flow itself; under such circumstances the size again becomes a significant parameter—though for the model bed alone.

In by far the majority of actual canals and rivers, the bed material is very small in comparison with the depth of flow, while the depth, mean velocity, and bed roughness are such that the motion is wholly turbulent, viscous effects upon the resistance then being quite negligible. At sufficiently advanced stages of sediment movement the sediment diameter probably influences the rate of transport only insofar as it governs the velocity of fall. In such cases, the fall velocity would seem the only logical sediment characteristic for similarity criteria, the ratio V/w_m and the factor σ_w being of basic importance; needless to say, however, the model scale should not be so far reduced as to prevent the full development of turbulence.

Experiments now in progress on an arbitrary condition of localized scour indicate that the mean fall velocity and the standard deviation about this mean are again significant, the ratio V/w_m being a primary similarity parameter so long as the model scale is not so small as to yield a grain diameter approaching the order of magnitude of other flow dimensions. In this connection it is to be noted that the customary use of coarse, low-density materials in model studies of scour produces a condition of transport akin to the bed-load phase of uniform flow, whereas in nature scour is often predominantly a phenomenon of suspension.

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